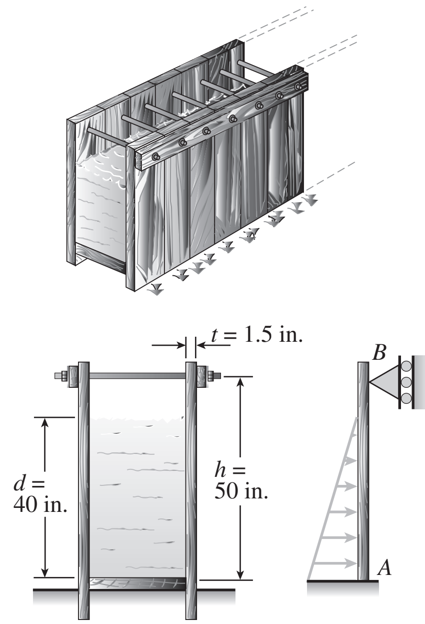
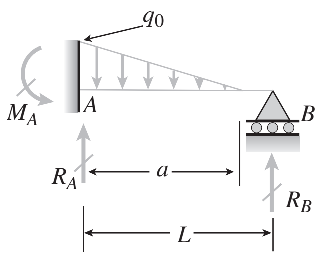
Problem 10.4-15 A temporary wood flume serving as a channel for irrigation water is shown in the figure. The vertical boards forming the sides of the flume are sunk in the ground, which provides a fixed support. The top of the flume is held by tie rods that are tightened so that there is no deflection of the boards at that point. Thus, the vertical boards may be modeled as a beam *AB*, supported and loaded as shown in the last part of the figure.

Assuming that the thickness *t* of the boards is 1.5 in., the depth *d* of the water is 40 in., and the height *h* to the tie rods is 50 in., what is the maximum bending stress σ in the boards? (*Hint:* The numerically largest bending moment occurs at the fixed support.)



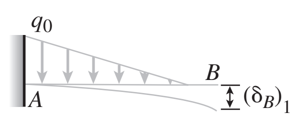
Solution 10.4-15 Side wall of a wood flume



Select R_B as redundant.

Equilibrium: $M_A = \dfrac{q_0 a^2}{6} - R_B L$

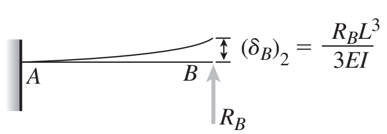
RELEASED STRUCTURE AND FORCE-DISPL. EQS.



From Table G-1, Case B:

$$(\delta_B)_1 = \frac{q_0 a^4}{30EI} + \frac{q_0 a^3}{24EI}(L - a) = \frac{q_0 a^3}{120EI}(5L - a)$$

$$(\delta_B)_2 = \frac{R_B L^3}{3EI}$$



COMPATIBILITY

$$\delta_B = (\delta_B)_1 - (\delta_B)_2 = 0 \quad \therefore R_B = \frac{q_0 a^3(5L - a)}{40L^3}$$

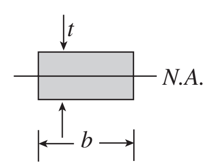
MAXIMUM BENDING MOMENT

$$M_{max} = M_A = \frac{1}{6} q_0 a^2 - R_B L$$

$$= \frac{q_0 a^2}{120L^2}(20L^2 - 15aL + 3a^2)$$

NUMERICAL VALUES

$a = 40$ in. $\quad L = 50$ in. $\quad t = 1.5$ in.
$b =$ width of beam



$$S = \frac{bt^2}{6} \quad \sigma = \frac{M_{max}}{S}$$

$\gamma = 62.4$ lb/ft^3 $= 0.03611$ lb/in.^{3}

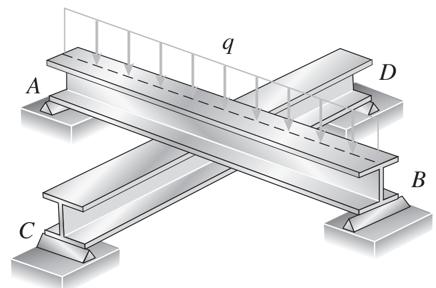
Pressure $p = \gamma a \quad q_0 = pb = \gamma ab$

$$M_{max} = \frac{\gamma a^3 b}{120L^2}(20L^2 - 15aL + 3a^2) = 191.05b$$

$$S = \frac{bt^2}{6} = 0.3750b \quad \sigma = \frac{M_{max}}{S} = 509 \text{ psi} \quad \longleftarrow$$

Problem 10.4-16 Two identical, simply supported beams *AB* and *CD* are placed so that they cross each other at their midpoints (see figure). Before the uniform load is applied, the beams just touch each other at the crossing point.

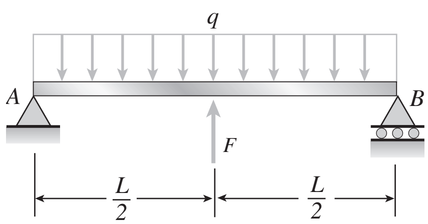
Determine the maximum bending moments $(M_{AB})_{max}$ and $(M_{CD})_{max}$ in beams *AB* and *CD*, respectively, due to the uniform load if the intensity of the load is $q = 6.4$ kN/m and the length of each beam is $L = 4$ m.



Solution 10.4-16 Two beams that cross

F = interaction force between the beams

UPPER BEAM



$(\delta_B)_1$ = downward deflection due to q

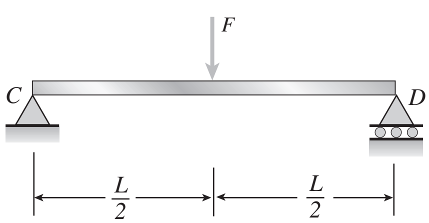
$$= \frac{5qL^4}{384EI}$$

$(\delta_B)_2$ = upward deflection due to F

$$= \frac{FL^3}{48EI}$$

$$\delta_{AB} = (\delta_B)_1 - (\delta_B)_2 = \frac{5qL^4}{384EI} - \frac{FL^3}{48EI}$$

LOWER BEAM

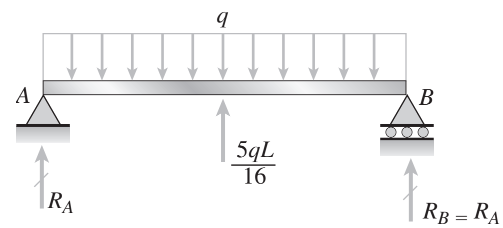


$$\delta_{CD} = \frac{FL^3}{48EI}$$

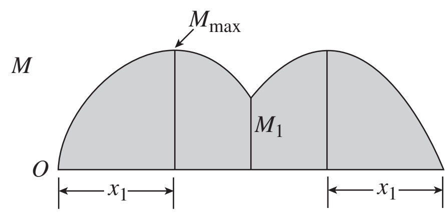
COMPATIBILITY $\delta_{AB} = \delta_{CD}$

$$\frac{5qL^4}{384EI} - \frac{FL^3}{48EI} = \frac{FL^3}{48EI} \quad \therefore F = \frac{5qL}{16}$$

UPPER BEAM



$$R_A = \frac{11qL}{32}$$



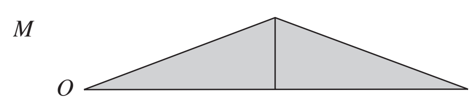
$$x_1 = \frac{11L}{32}$$

$$M_{max} = \frac{121qL^2}{2048}$$

$$M_1 = \frac{3qL^2}{64} \qquad (M_{AB})_{max} = \frac{121qL^2}{2048} \quad \leftarrow$$

LOWER BEAM

$$M_{max} = \frac{FL}{4} = \frac{5qL^2}{64}$$



$$(M_{CD})_{max} = \frac{5qL^2}{64} \quad \leftarrow$$

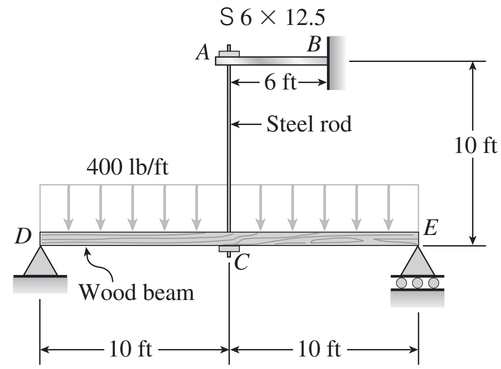
NUMERICAL VALUES

$q = 6.4$ kN/m $\quad (M_{AB})_{max} = 6.05$ kN · m $\leftarrow$

$L = 4$ m $\quad (M_{CD})_{max} = 8.0$ kN · m $\leftarrow$

Problem 10.4-17 The cantilever beam AB shown in the figure is an S 6 × 12.5 steel I-beam with $E = 30 \times 10^6$ psi. The simple beam DE is a wood beam 4 in. × 12 in. (nominal dimensions) in cross section with $E = 1.5 \times 10^6$ psi. A steel rod AC of diameter 0.25 in., length 10 ft, and $E = 30 \times 10^6$ psi serves as a hanger joining the two beams. The hanger fits snugly between the beams before the uniform load is applied to beam DE.

Determine the tensile force F in the hanger and the maximum bending moments M_{AB} and M_{DE} in the two beams due to the uniform load, which has intensity $q = 400$ lb/ft. (*Hint:* To aid in obtaining the maximum bending moment in beam DE, draw the shear-force and bending-moment diagrams.)

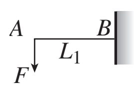


Solution 10.4-17 **Beams joined by a hanger**

F = tensile force in hanger

Select F as redundant.

(1) CANTILEVER BEAM AB



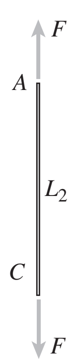
S 6 × 12.5 $I_1 = 22.1 \text{ in.}^4$

$L_1 = 6 \text{ ft} = 72 \text{ in.}$

$E_1 = 30 \times 10^6 \text{ psi}$

$$(\delta_A)_1 = \frac{FL_1^3}{3E_1I_1} = 187.66 \times 10^{-6}F \quad \begin{cases} F = \text{lb} \\ \delta = \text{in.} \end{cases}$$

(2) HANGER AC



$d = 0.25 \text{ in.}$ $L_2 = 10 \text{ ft} = 120 \text{ in.}$

$E_2 = 30 \times 10^6 \text{ psi}$

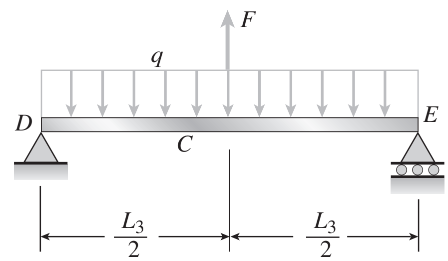
$$A_2 = \frac{\pi d^2}{4} = 0.049087 \text{ in.}^2$$

Δ = elongation of AC

$$\Delta = \frac{FL_2}{E_2A_2} = 81.488 \times 10^{-6}F$$

(F = lb, Δ = in.)

(3) BEAM DCE



$L_3 = 20 \text{ ft} = 240 \text{ in.}$

$q = 400 \text{ lb/ft}$

$= 33.333 \text{ lb/in.}$

$E_3 = 1.5 \times 10^6 \text{ psi}$

4 in. × 12 in. (nominal)

$I_3 = 415.28 \text{ in.}^4$

$$(\delta_C)_3 = \frac{5qL_3^4}{384E_3I_3} - \frac{FL_3^3}{48E_3I_3}$$

$$= 2.3117 \text{ in.} - 462.34 \times 10^{-6}\,F \quad \begin{cases} F = \text{lb} \\ \delta = \text{in.} \end{cases}$$

COMPATIBILITY

$(\delta_A)_1 + \Delta = (\delta_C)_3$

$187.66 \times 10^{-6}\,F + 81.488 \times 10^{-6}\,F$

$= 2.3117 - 462.34 \times 10^{-6}\,F$

$F = 3160 \text{ lb}$ ←

(1) MAX. MOMENT IN AB

$M_{AB} = FL_1 = (3160 \text{ lb})(6 \text{ ft})$

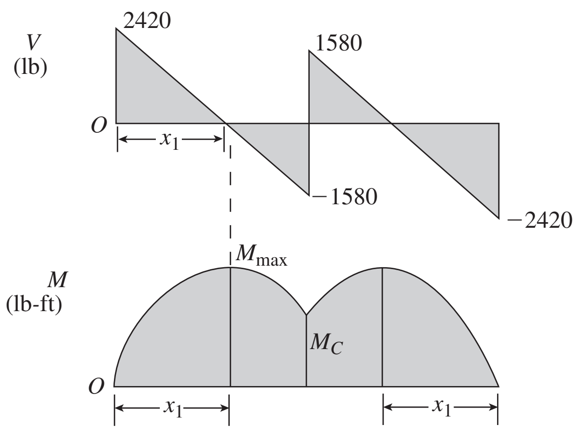
$= 18{,}960 \text{ lb-ft}$ ←

(3) MAX. MOMENT IN DCE

$$R_D = \frac{qL_3}{2} - \frac{F}{2} = 2420 \text{ lb}$$

(Continued)

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS



$x_1 = 6.050$ ft

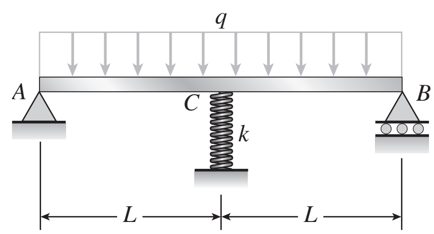
$M_{max} = 7320$ lb-ft

$M_C = 4200$ lb-ft

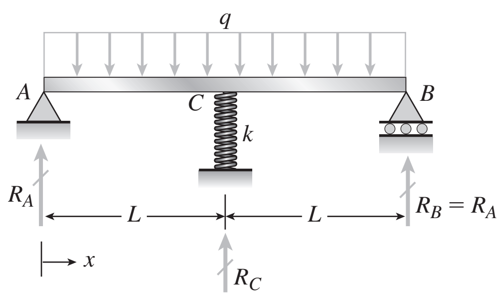
$M_{DE} = 7320$ lb-ft ←

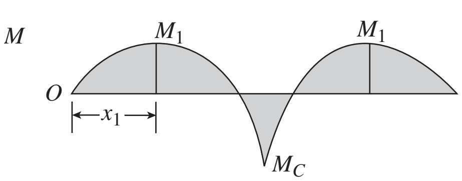
Problem 10.4-18 The beam AB shown in the figure is simply supported at A and B and supported on a spring of stiffness k at its midpoint C. The beam has flexural rigidity EI and length $2L$.

What should be the stiffness k of the spring in order that the maximum bending moment in the beam (due to the uniform load) will have the smallest possible value?



Solution 10.4-18 **Beam supported by a spring**





BENDING MOMENT $M = R_A x - \dfrac{qx^2}{2}$

LOCATION OF MAXIMUM POSITIVE MOMENT

$$\frac{dM}{dx} = 0 \quad R_A - qx = 0 \quad x_1 = \frac{R_A}{q}$$

MAXIMUM POSITIVE MOMENT

$$M_1 = (M)_{x=x_1} = \frac{R_A^2}{2q}$$

MAXIMUM NEGATIVE MOMENT

$$M_C = (M)_{x=L} = R_A L - \frac{qL^2}{2}$$

FOR THE SMALLEST MAXIMUM MOMENT:

$|M_1| = |M_C|$ or $M_1 = -M_C$

$$\frac{R_A^2}{2q} = -R_A L + \frac{qL^2}{2}$$

Solve for R_A:

$$R_A = qL(\sqrt{2} - 1)$$

EQUILIBRIUM

$$\sum F_{vert} = 0 \quad 2R_A + R_C - 2qL = 0$$

$$R_C = 2\,qL\,(2 - \sqrt{2})$$

DOWNWARD DEFLECTION OF BEAM

$$(\delta_C)_1 = \frac{5\,qL^4}{24\,EI} - \frac{R_C L^3}{6EI} = \frac{qL^4}{24\,EI}(8\sqrt{2} - 11)$$

DOWNWARD DISPLACEMENT OF SPRING

$$(\delta_C)_2 = \frac{R_C}{k} = \frac{2\,qL}{k}(2 - \sqrt{2})$$

COMPATIBILITY $(\delta_C)_1 = (\delta_C)_2$

Solve for k:

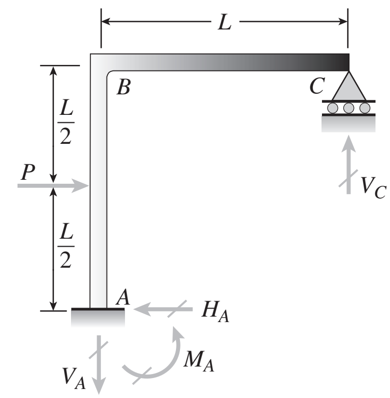
$$k = \frac{48\,EI}{7\,L^3}(6 + 5\sqrt{2})$$

$$= 89.63\frac{EI}{L^3} \quad \longleftarrow$$

Problem 10.4-19 The continuous frame ABC has a fixed support at A, a roller support at C, and a rigid corner connection at B (see figure). Members AB and BC each have length L and flexural rigidity EI. A horizontal force P acts at midheight of member AB.

(a) Find all reactions of the frame.

(b) What is the largest bending moment M_{max} in the frame? (*Note:* Disregard axial deformations in member AB and consider only the effects of bending.)



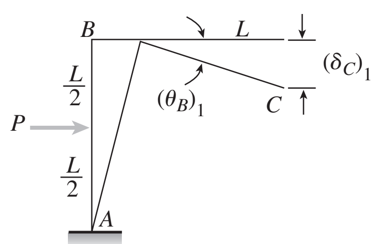
Solution 10.4-19 **Frame *ABC* with fixed support**

Select V_C as redundant.

EQUILIBRIUM $V_A = V_C \quad H_A = P$

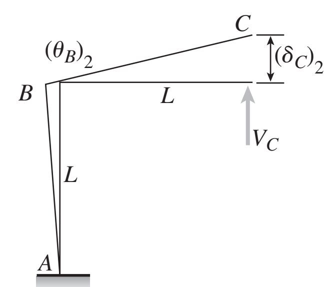
$M_A = PL/2 - V_C L$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$$(\theta_B)_1 = \frac{PL^2}{8EI}$$

$$(\delta_C)_1 = (\theta_B)_1 L = \frac{PL^3}{8EI}$$



$$(\theta_B)_2 = \frac{V_C L^2}{EI}$$

$$(\delta_C)_2 = (\theta_B)_2 L + \frac{V_C L^3}{3EI} = \frac{4V_C L^3}{3EI}$$

COMPATIBILITY $(\delta_C)_1 = (\delta_C)_2$

Substitute for $(\delta_C)_1$ and $(\delta_C)_2$ and solve:

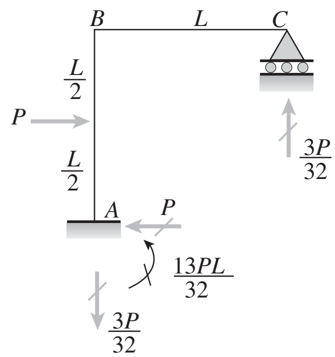
$$V_C = \frac{3P}{32} \quad \longleftarrow$$

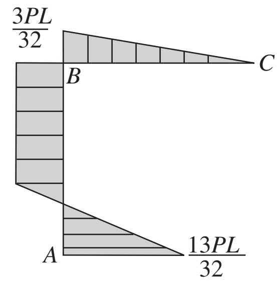
FROM EQUILIBRIUM:

$$V_A = \frac{3P}{32} \quad H_A = P \quad M_A = \frac{13PL}{32} \quad \longleftarrow$$

(Continued)

REACTIONS AND BENDING MOMENTS



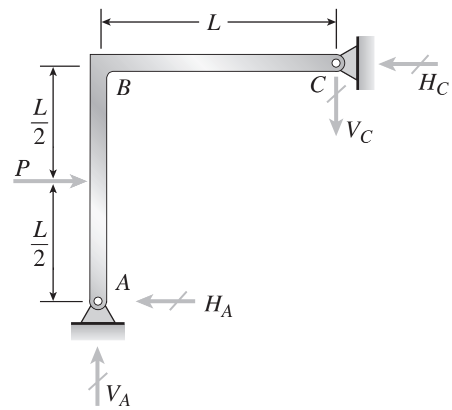


$$M_{max} = \frac{13PL}{32} \quad \longleftarrow$$

Problem 10.4-20 The continuous frame ABC has a pinned support at A, a pinned support at C, and a rigid corner connection at B (see figure). Members AB and BC each have length L and flexural rigidity EI. A horizontal force P acts at midheight of member AB.

(a) Find all reactions of the frame.

(b) What is the largest bending moment M_{max} in the frame? (*Note:* Disregard axial deformations in members AB and BC and consider only the effects of bending.)



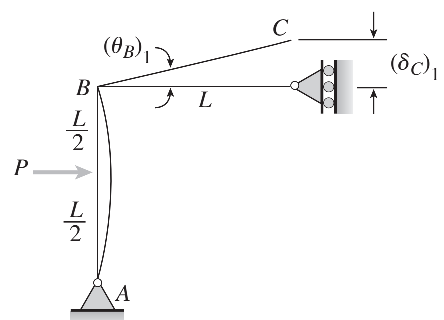
Solution 10.4-20 **Frame ABC with pinned supports**

Select V_C as redundant.

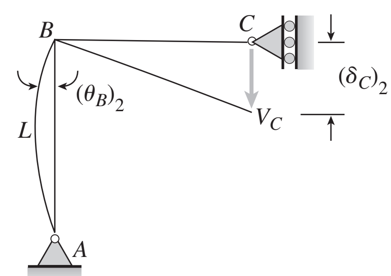
EQUILIBRIUM $V_A = V_C \quad H_A = \frac{P}{2} - V_C$

$$H_C = \frac{P}{2} + V_C$$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$$(\theta_B)_1 = \frac{PL^2}{16EI} \qquad (\delta_C)_1 = (\theta_B)_1 L = \frac{PL^3}{16EI}$$



$$(\theta_B)_2 = (V_C L)\frac{L}{3EI} = \frac{V_C L^2}{3EI}$$

$$(\delta_C)_2 = (\theta_B)_2 L + \frac{V_C L^3}{3EI} = \frac{2V_C L^3}{3EI}$$

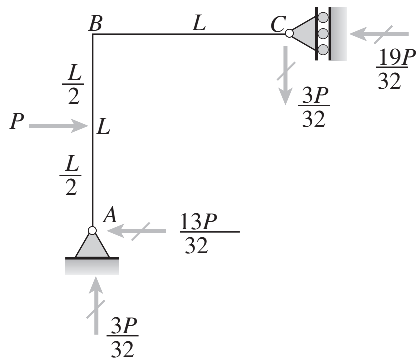
COMPATIBILITY

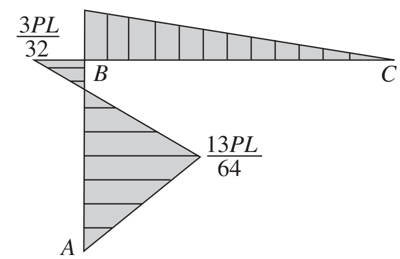
$$(\delta_C)_1 = (\delta_C)_2 \qquad \frac{PL^3}{16EI} = \frac{2V_C L^3}{3EI} \qquad V_C = \frac{3P}{32} \quad \longleftarrow$$

FROM EQUILIBRIUM:

$$V_A = \frac{3P}{32} \qquad H_A = \frac{13P}{32} \qquad H_C = \frac{19P}{32} \quad \longleftarrow$$

REACTIONS AND BENDING MOMENTS

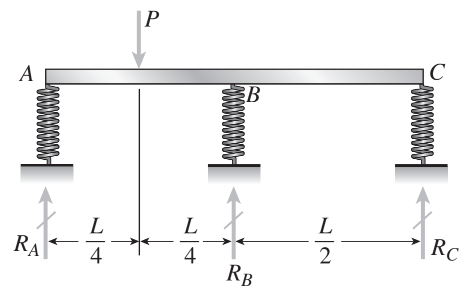




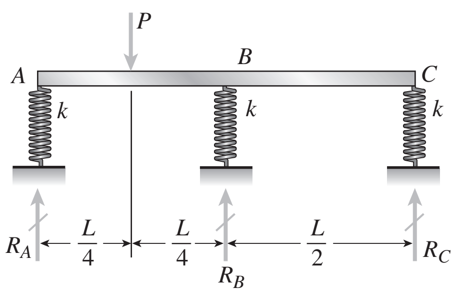
$$M_{\max} = \frac{13\,PL}{64}$$

Problem 10.4-21 A wide-flange beam ABC rests on three identical spring supports at points A, B, and C (see figure). The flexural rigidity of the beam is $EI = 6912 \times 10^6$ lb-in.^{2}, and each spring has stiffness $k = 62{,}500$ lb/in. The length of the beam is $L = 16$ ft.

If the load P is 6000 lb, what are the reactions R_A, R_B, and R_C? Also, draw the shear-force and bending-moment diagrams for the beam, labeling all critical ordinates.



Solution 10.4-21 **Beam on three springs**

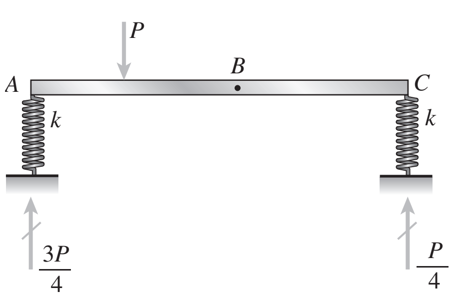


Select R_B as redundant.

EQUILIBRIUM

$$R_A = \frac{3P}{4} - \frac{R_B}{2} \qquad R_C = \frac{P}{4} - \frac{R_B}{2}$$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



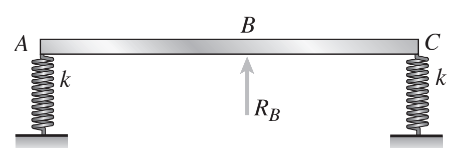
$$(\delta_A)_1 = \frac{3P}{4k}$$

$$(\delta_C)_1 = \frac{P}{4k}$$

$$(\delta_B)_1 = \frac{1}{2}[(\delta_A)_1 + (\delta_C)_1] + \frac{P\left(\frac{L}{4}\right)\left[3L^2 - 4\left(\frac{L}{4}\right)^2\right]}{48EI}$$

(Case 5, Table G-2)

$$(\delta_B)_1 = \frac{P}{2k} + \frac{11\,PL^3}{768EI} \qquad \text{(downward)}$$



$$(\delta_A)_2 = \frac{R_B}{2k}$$

$$(\delta_C)_2 = \frac{R_B}{2k}$$

$$(\delta_B)_2 = \frac{1}{2}[(\delta_A)_2 + (\delta_C)_2] + \frac{R_B L^3}{48\,EI}$$

$$= \frac{R_B}{2k} + \frac{R_B L^3}{48\,EI} \qquad \text{(upward)}$$

(Continued)

COMPATIBILITY $(\delta_B)_1 - (\delta_B)_2 = \dfrac{R_B}{k}$

Substitute and solve:

$$R_B = P\left(\frac{384\,EI + 11kL^3}{1152\,EI + 16\,kL^3}\right)$$

Let $k^* = \dfrac{kL^3}{EI}$ (nondimensional) ←

$$R_B = \frac{P}{16}\left(\frac{384 + 11k^*}{72 + k^*}\right) \quad \leftarrow$$

FROM EQUILIBRIUM:

$$R_A = \frac{P}{32}\left(\frac{1344 + 13k^*}{72 + k^*}\right) \quad \leftarrow$$

$$R_C = \frac{3P}{32}\left(\frac{64 - k^*}{72 + k^*}\right) \quad \leftarrow$$

NUMERICAL VALUES

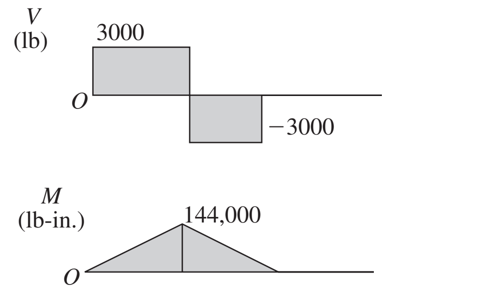
$EI = 6912 \times 10^6$ lb-in.^{2} $\quad k = 62{,}500$ lb/in.

$L = 16$ ft $= 192$ in. $\quad P = 6000$ lb

$k^* = \dfrac{kL^3}{EI} = 64 \quad R_B = 3000$ lb ←

$R_A = 3000$ lb $\quad R_C = 0$ ←

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS



Problem 10.4-22 A fixed-end beam AB of length L is subjected to a uniform load of intensity q acting over the middle region of the beam (see figure).

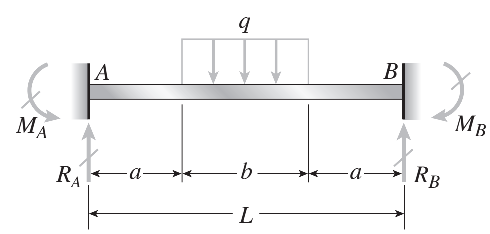
(a) Obtain a formula for the fixed-end moments M_A and M_B in terms of the load q, the length L, and the length b of the loaded part of the beam.

(b) Plot a graph of the fixed-end moment M_A versus the length b of the loaded part of the beam. For convenience, plot the graph in the following nondimensional form:

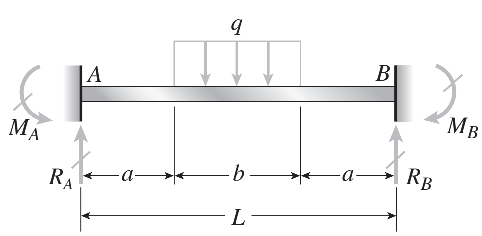
$$\frac{M_A}{qL^2/12} \quad \text{versus} \quad \frac{b}{L}$$

with the ratio b/L varying between its extreme values of 0 and 1.

(c) For the special case in which $a = b = L/3$, draw the shear-force and bending-moment diagrams for the beam, labeling all critical ordinates.



Solution 10.4-22 Fixed-end beam

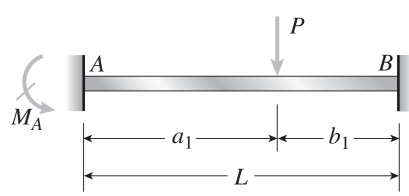


$M_A = M_B$

$$R_A = R_B = \frac{qb}{2}$$

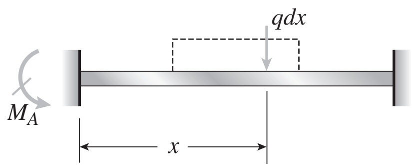
$$a = \frac{L - b}{2}$$

FROM EXAMPLE 10-4, EQ. (10-25a):



$$M_A = \frac{Pa_1b_1^2}{L^2}$$

For the partial uniform load



$$dM_A = \frac{(qdx)(x)(L-x)^2}{L^2}$$

$$M_A = \int_a^{a+b} dM_A = \int_{(L-b)/2}^{(L+b)/2} dM_A$$

$$= \frac{q}{L^2}\int_{(L-b)/2}^{(L+b)/2} x(L-x)^2 dx$$

$$= \frac{q}{L^2}\int_{(L-b)/2}^{(L+b)/2} (L^2x - 2Lx^2 + x^3)\, dx$$

$$= \frac{q}{L^2}\left[\frac{L^2x^2}{2} - \frac{2Lx^3}{3} + \frac{x^4}{4}\right]_{(L-b)/2}^{(L+b)/2}$$

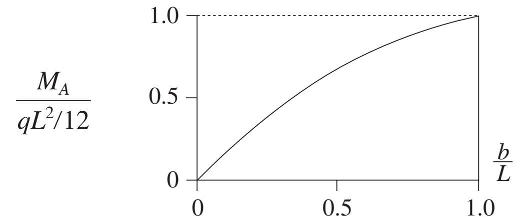
. . . (lengthy substitution) . . .

$$= \frac{qb}{24L}(3L^2 - b^2)$$

(a) $M_A = M_B = \dfrac{qb}{24L}(3L^2 - b^2)$ ⟵

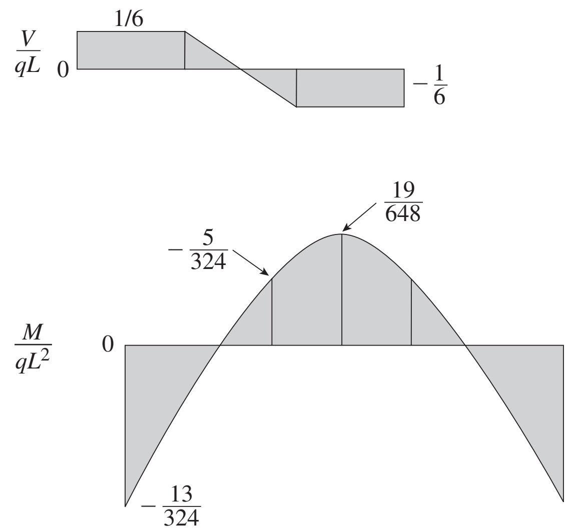
(b) Graph of fixed-end moment

$$\frac{M_A}{qL^2/12} = \frac{b}{2L}\left(3 - \frac{b^2}{L^2}\right)$$



(c) Special case $a = b = L/3$

$$R_A = R_B = \frac{qL}{6} \qquad M_A = M_B = \frac{13qL^2}{324}$$

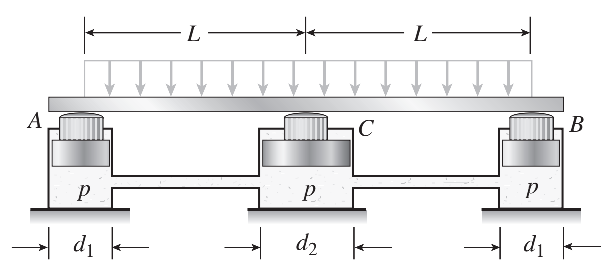


Problem 10.4-23 A beam supporting a uniform load of intensity q throughout its length rests on pistons at points A, C, and B (see figure). The cylinders are filled with oil and are connected by a tube so that the oil pressure on each piston is the same. The pistons at A and B have diameter d_1, and the piston at C has diameter d_2.

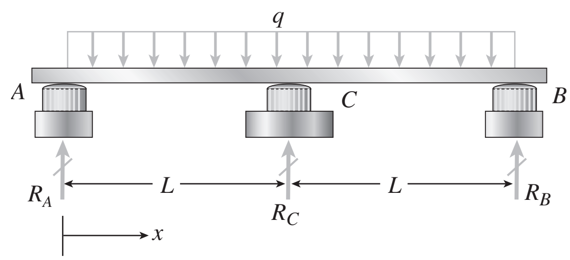
(a) Determine the ratio of d_2 to d_1 so that the largest bending moment in the beam is as small as possible.

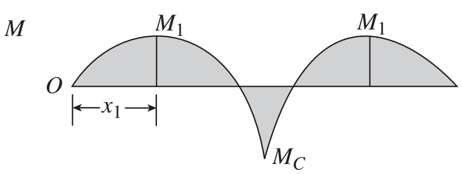
(b) Under these optimum conditions, what is the largest bending moment M_{max} in the beam?

(c) What is the difference in elevation between point C and the end supports?



Solution 10.4-23 Beam supported by pistons





BENDING MOMENT $\quad M = R_A x - \dfrac{qx^2}{2}$

LOCATION OF MAXIMUM POSITIVE MOMENT

$$\frac{dM}{dx} = 0 \qquad R_A - qx = 0 \qquad x_1 = \frac{R_A}{q}$$

MAXIMUM POSITIVE MOMENT

$$M_1 = (M)_{x=x_1} = \frac{R_A^2}{2q}$$

MAXIMUM NEGATIVE MOMENT

$$M_C = (M)_{x=L} = R_A L - \frac{qL^2}{2}$$

FOR THE SMALLEST MAXIMUM MOMENT:

$|M_1| = |M_C|$ OR $M_1 = -M_C$

$$\frac{R_A^2}{2q} = -R_A L + \frac{qL^2}{2}$$

Solve for R_A: $\quad R_A = qL(\sqrt{2} - 1)$

EQUILIBRIUM

$$\sum F_{\text{vert}} = 0 \qquad 2R_A + R_C - 2qL = 0$$
$$R_C = 2qL(2 - \sqrt{2})$$

REACTIONS BASED UPON PRESSURE

$$R_A = R_B = p\left(\frac{\pi d_1^2}{4}\right) \qquad R_C = p\left(\frac{\pi d_2^2}{4}\right)$$

(a) $\quad \therefore \dfrac{d_2}{d_1} = \sqrt{\dfrac{R_C}{R_A}} = \sqrt{\dfrac{2(2-\sqrt{2})}{\sqrt{2}-1}} = \sqrt[4]{8}$

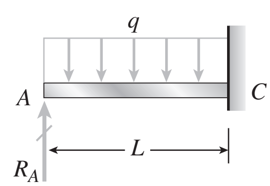
$= 1.682$ ←

(b) $\quad M_{\text{MAX}} = M_1 = \dfrac{R_A^2}{2q} = \dfrac{qL^2}{2}(3 - 2\sqrt{2})$

$= 0.08579\, qL^2$ ←

(c) DIFFERENCE IN ELEVATION

By symmetry, beam has zero slope at C.



$R_A = qL(\sqrt{2} - 1)$

δ_A = Difference in elev.

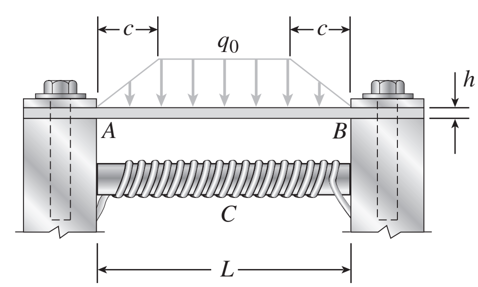
$$\delta_A = \frac{R_A L^3}{3EI} - \frac{qL^4}{8EI} = \frac{qL^4}{24EI}(8\sqrt{2} - 11)$$

$= 0.01307\, qL^4/EI$ ←

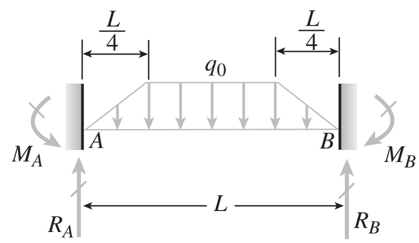
Point C is below points A and B by the amount $0.01307qL^4/EI$.

Problem 10.4-24 A thin steel beam AB used in conjunction with an electromagnet in a high-energy physics experiment is securely bolted to rigid supports (see figure). A magnetic field produced by coils C results in a force acting on the beam. The force is trapezoidally distributed with maximum intensity $q_0 = 18$ kN/m. The length of the beam between supports is $L = 200$ mm and the dimension c of the trapezoidal load is 50 mm. The beam has a rectangular cross section with width $b = 60$ mm and height $h = 20$ mm.

Determine the maximum bending stress $\sigma_{\max}$ and the maximum deflection $\delta_{\max}$ for the beam. (Disregard any effects of axial deformations and consider only the effects of bending. Use $E = 200$ GPa.)



Solution 10.4-24 **Fixed-end beam (trapezoidal load)**



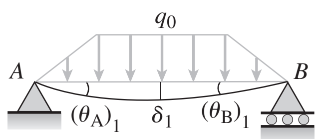
$c = \dfrac{L}{4}$

From symmetry and equilibrium

$$M_A = M_B \qquad R_A = R_B = \frac{3q_0L}{8}$$

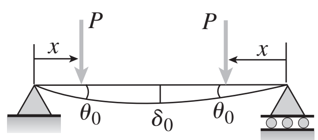
Select M_A and M_B as redundants

Released structure with applied load



$(\theta_A)_1 = (\theta_B)_1$

Consider the following beam from Case 6, Table G-2:



$$\theta_0 = \frac{Px(L-x)}{2EI} \qquad \delta_0 = \frac{Px}{24EI}(3L^2 - 4x^2)$$

Consider the load P as an element of the distributed load.

Replace P by qdx, where

$$q = \frac{4q_0x}{L} \qquad x \text{ from 0 to } L/4$$

$$q = q_0 \qquad x \text{ from } L/4 \text{ to } L/2$$

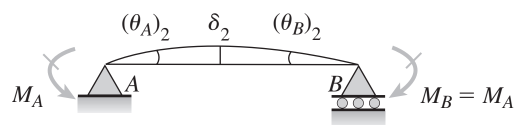
$$(\theta_A)_1 = \frac{1}{2EI}\int_0^{L/4}\left(\frac{4q_0x}{L}\right)(x)(L-x)\,dx + \frac{1}{2EI}\int_{L/4}^{L/2} q_0x(L-x)\,dx$$

$$= \frac{13q_0L^3}{1536\,EI} + \frac{11q_0L^3}{384\,EI} = \frac{19q_0L^3}{512EI}$$

$$\delta_1 = \frac{1}{24EI}\int_0^{L/4}\left(\frac{4q_0x}{L}\right)(x)(3L^2 - 4x^2)\,dx + \frac{1}{24EI}\int_{L/4}^{L/2} q_0x(3L^2 - 4x^2)\,dx$$

$$= \frac{19q_0L^4}{7680EI} + \frac{19q_0L^4}{2048EI} = \frac{361q_0L^4}{30{,}720EI}$$

Released structure with redundants



$$(\theta_A)_2 = (\theta_B)_2 \qquad M_B = M_A$$

From Case 10, Table G-2:

$$(\theta_A)_2 = \frac{M_AL}{2EI} \qquad \delta_2 = \frac{M_AL^2}{8EI}$$

Compatibility

$$\theta_A = (\theta_A)_1 - (\theta_A)_2 = 0$$

$$\frac{19\,q_0L^3}{512\,EI} - \frac{M_AL}{2\,EI} = 0 \qquad M_A = \frac{19q_0L^2}{256}$$

Deflection at the midpoint

$$\delta_{\max} = \delta_1 - \delta_2 = \frac{361q_0L^4}{30{,}720\,EI} - \frac{M_AL^2}{8\,EI}$$

$$= \frac{361q_0L^4}{30{,}720\,EI} - \left(\frac{19q_0L^2}{256}\right)\left(\frac{L^2}{8\,EI}\right)$$

$$= \frac{19q_0L^4}{7680\,EI}$$

Bending moment at the midpoint

$$M_C = R_A\left(\frac{L}{2}\right) - M_A - \frac{q_0L^2}{24} - \frac{q_0L^2}{32}$$

$$= \frac{3q_0L}{8}\left(\frac{L}{2}\right) - \frac{19q_0L^2}{256} - \frac{7q_0L^2}{96} = \frac{31q_0L^2}{768}$$

Maximum bending moment

$$M_A > M_C \qquad \therefore\ M_{\max} = M_A = \frac{19q_0L^2}{256}$$

(Continued)

NUMERICAL VALUES

$q_0 = 18$ kN/m $\quad L = 200$ mm $\quad b = 60$ mm

$h = 20$ mm $\quad E = 200$ GPa

$$S = \frac{bh^2}{6} = 4.0 \times 10^{-6}\ \text{m}^3$$

$$I = \frac{bh^3}{12} = 40 \times 10^{-9}\ \text{m}^4$$

$$M_{\max} = \frac{19\,q_0 L^2}{256} = 53.44\ \text{N} \cdot \text{m}$$

$$\sigma_{\max} = \frac{M_{\max}}{S} = 13.4\ \text{MPa} \longleftarrow$$

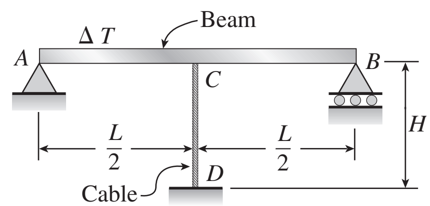
$$\delta_{\max} = \frac{19\,q_0 L^4}{7680\,EI} = 0.00891\ \text{mm} \longleftarrow$$

Temperature Effects

The beams described in the problems for Section 10.5 have constant flexural rigidity EI.

Problem 10.5-1 A cable CD of length H is attached to the midpoint of a simple beam AB of length L (see figure). The moment of inertia of the beam is I, and the effective cross-sectional area of the cable is A. The cable is initially taut but without any initial tension.

Obtain a formula for the tensile force S in the cable when the temperature drops uniformly by ΔT degrees, assuming that the beam and cable are made of the same material (modulus of elasticity E and coefficient of thermal expansion α). (Use the method of superposition in the solution.)



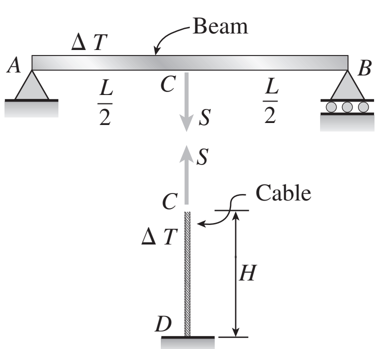
Solution 10.5-1 Uniform temperature change

$\Delta T =$ Decrease in temperature

Use method of superposition.

Select tensile force S in the cable as redundant.

RELEASED STRUCTURE



$I =$ Moment of inertia

$A =$ Cross-sectional area

BEAM $\quad (\delta_C)_1 = \dfrac{SL^3}{48\,EI}$ (downward)

CABLE $\quad (\delta_C)_2 = \alpha H(\Delta T) - \dfrac{SH}{EA}$ (downward)

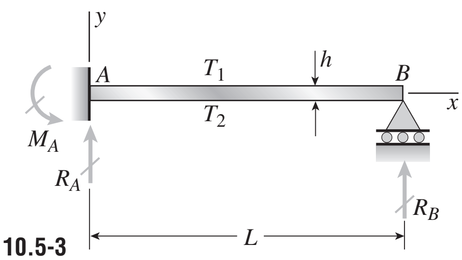
COMPATIBILITY $\quad (\delta_C)_1 = (\delta_C)_2$

$$\frac{SL^3}{48\,EI} = \alpha H(\Delta T) - \frac{SH}{EA}$$

SOLVE FOR S: $\quad S = \dfrac{48\,EIAH\alpha(\Delta T)}{AL^3 + 48\,IH} \longleftarrow$

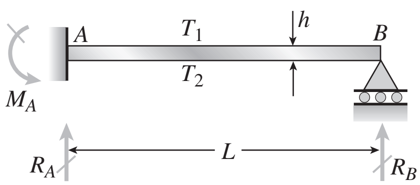
Problem 10.5-2 A propped cantilever beam, fixed at the left-hand end A and simply supported at the right-hand end B, is subjected to a temperature differential with temperature T_1 on its upper surface and T_2 on its lower surface (see figure).

Find all reactions for this beam. (Use the method of superposition in the solution. Also, if desired, use the results from Problem 9.13-1.)



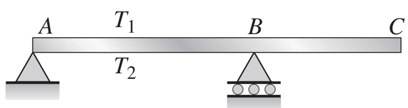
Probs. 10.5-2 and 10.5-3

Solution 10.5-2 **Beam with temperature differential**



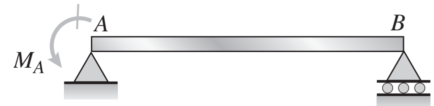
Use the method of superposition.
Select M_A as redundant.

RELEASED STRUCTURE



$$(\theta_A)_1 = \frac{\alpha L(T_2 - T_1)}{2h} \quad \text{(clockwise)}$$

(From the answer to Prob. 9.11-1)



$$(\theta_A)_2 = \frac{M_A L}{3EI} \quad \text{(counterclockwise)}$$

COMPATIBILITY $(\theta_A)_1 = (\theta_A)_2$

$$\frac{\alpha L(T_2 - T_1)}{2h} = \frac{M_A L}{3EI} \qquad M_A = \frac{3\alpha EI(T_2 - T_1)}{2h} \longleftarrow$$

EQUILIBRIUM

$$\sum M_B = 0 \qquad M_A - R_A L = 0$$

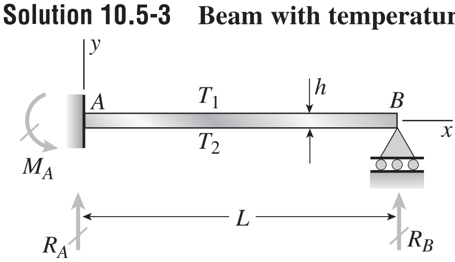
$$R_A = \frac{3\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$$

$$\sum F_{\text{vert}} = 0 \qquad R_B = -R_A$$

$$R_B = -\frac{3\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$$

Problem 10.5-3 Solve the preceding problem by integrating the differential equation of the deflection curve.

Solution 10.5-3 **Beam with temperature differential**



$$M = R_B(L - x)$$

DIFFERENTIAL EQUATION (EQ. 10-39b)

$$EIv'' = M + \frac{\alpha EI(T_2 - T_1)}{h}$$

$$\text{or} \quad EIv'' = R_B(L - x) + \frac{\alpha EI(T_2 - T_1)}{h}$$

$$EIv' = R_B Lx - R_B\left(\frac{x^2}{2}\right) + \frac{\alpha EI(T_2 - T_1)}{h}x + C_1$$

B.C. 1 $v'(0) = 0 \quad \therefore C_1 = 0$

$$EIv = R_B L\left(\frac{x^2}{2}\right) - R_B\left(\frac{x^3}{6}\right) + \frac{\alpha EI(T_2 - T_1)}{2h}x^2 + C_2$$

B.C. 2 $v(0) = 0 \quad \therefore C_2 = 0$

B.C. 3 $v(L) = 0$

$$\therefore R_B = -\frac{3\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$$

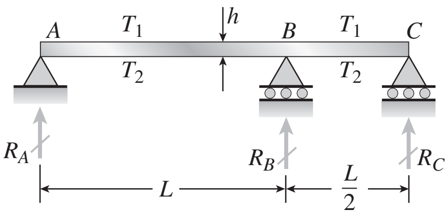
FROM EQUILIBRIUM:

$$R_A = -R_B = \frac{3\alpha EI(T_2 - T_1)}{2hL} \longleftarrow$$

$$M_A = R_A L \qquad M_A = \frac{3\alpha EI(T_2 - T_1)}{2h} \longleftarrow$$

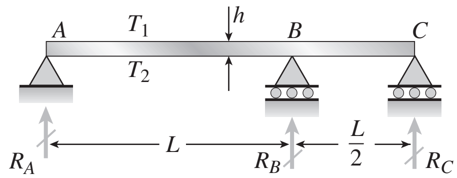
Problem 10.5-4 A two-span beam with spans of lengths L and $L/2$ is subjected to a temperature differential with temperature T_1 on its upper surface and T_2 on its lower surface (see figure).

Determine all reactions for this beam. (Use the method of superposition in the solution. Also, if desired, use the results from Problems 9.8-5 and 9.13-3.)



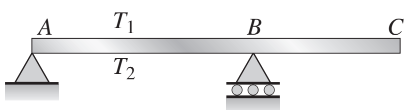
Probs. 10.5-4 and 10.5-5

Solution 10.5-4 Beam with temperature differential



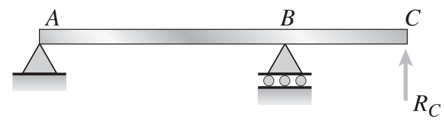
Use the method of superposition.
Select R_C as redundant.

RELEASED STRUCTURE



From Prob. 9.13-3:

$$(\delta_C)_1 = \frac{3\alpha L^2 (T_2 - T_1)}{8h} \text{ (upward)}$$



From Prob. 9.8-5:

$$(\delta_C)_2 = \frac{R_C L^3}{8EI} \text{ (upward)}$$

COMPATIBILITY $\quad (\delta_C)_1 + (\delta_C)_2 = 0$

$$\frac{3\alpha L^2(T_2 - T_1)}{8h} = -\frac{R_C L^3}{8EI}$$

$$R_C = -\frac{3\alpha EI(T_2 - T_1)}{hL} \quad \longleftarrow$$

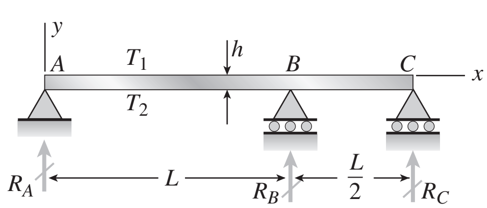
FROM EQUILIBRIUM:

$$R_A = \frac{R_C}{2} \qquad R_A = -\frac{3\alpha EI(T_2 - T_1)}{2hL} \quad \longleftarrow$$

$$R_B = -\frac{3R_C}{2} \qquad R_B = \frac{9\alpha EI(T_2 - T_1)}{2hL} \quad \longleftarrow$$

Problem 10.5-5 Solve the preceding problem by integrating the differential equation of the deflection curve.

Solution 10.5-5 Beam with temperature differential



DIFFERENTIAL EQUATION (EQ. 10-39b)

$$EIv'' = M + \frac{\alpha EI(T_2 - T_1)}{h}$$

For convenience, let $\beta = \dfrac{\alpha EI(T_2 - T_1)}{h}$ (1)

$EIv'' = M + \beta$ (2)

PART AB OF THE BEAM $(0 \leq x \leq L)$

$M = R_A x \qquad EIv'' = R_A x + \beta$

$EIv' = R_A x^2/2 + \beta x + C_1$ (3)

$EIv = R_A x^3/6 + \beta x^2/2 + C_1 x + C_2$ (4)

B.C. 1 $\quad v(0) = 0 \qquad \therefore C_2 = 0$

B.C. 2 $\quad v(L) = 0 \qquad \therefore R_A L^2 + 6C_1 = -3\beta L$ (5)

PART BC OF THE BEAM ($L \le x \le 3L/2$)

$M = R_A x + R_B(x - L)$

From equilibrium, $R_B = -3R_A$ (6)

$\therefore M = -2R_A x + 3R_A L$

$EIv'' = M + \beta = -2R_A x + 3R_A L + \beta$

$EIv' = -R_A x^2 + 3R_A Lx + \beta x + C_3$ (7)

$EIv = -R_A x^3/3 + 3R_A Lx^2/2 + \beta x^2/2 + C_3 x + C_4$ (8)

B.C. 3 $v(L) = 0$

$\therefore 7R_A L^3 + 6C_3 L + 6C_4 = -3\beta L^2$ (9)

B.C. 4 $v(3L/2) = 0$

$\therefore 18R_A L^3 + 12C_3 L + 8C_4 = -9\beta L^2$ (10)

CONTINUITY CONDITION AT B

$(EIv')_{AB} = (EIv')_{BC}$ at $x = L$

From Eqs. (3) and (7):

$R_A(L^2/2) + \beta L + C_1 = -R_A L^2 + 3R_A L^2 + \beta L + C_3$

or $3R_A L^2 - 2C_1 + 2C_3 = 0$ (11)

SOLVE EQS. (5), (9), (10), AND (11) FOR R_A:

$$R_A = -\frac{3\beta}{2L} = -\frac{3\alpha EI(T_2 - T_1)}{2hL} \quad \longleftarrow$$

Also: $C_1 = -\beta L/4$ $C_2 = 0$ $C_3 = 2\beta L$

$C_4 = -3\beta L^2/4$

From Eq. (6): $$R_B = \frac{9\alpha EI(T_2 - T_1)}{2hL} \quad \longleftarrow$$

From equilibrium:

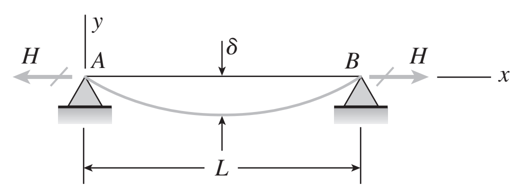
$$R_C = 2R_A = -\frac{3\alpha EI(T_2 - T_1)}{hL} \quad \longleftarrow$$

Longitudinal Displacements at the Ends of Beams

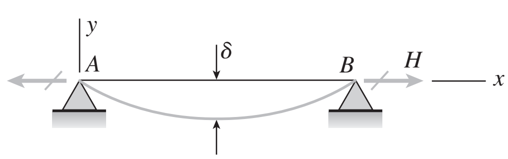
Problem 10.6-1 Assume that the deflected shape of a beam AB with *immovable* pinned supports (see figure) is given by the equation $v = -\delta \sin \pi x/L$, where δ is the deflection at the midpoint of the beam and L is the length. Also, assume that the beam has constant axial rigidity EA.

(a) Obtain formulas for the longitudinal force H at the ends of the beam and the corresponding axial tensile stress σ_t.

(b) For an aluminum-alloy beam with $E = 10 \times 10^6$ psi, calculate the tensile stress σ_t when the ratio of the deflection δ to the length L equals 1/200, 1/400, and 1/600.



Solution 10.6-1 **Beam with immovable supports**



(a)

$$v = -\delta \sin\frac{\pi x}{L} \qquad \frac{dv}{dx} = -\frac{\pi\delta}{L}\cos\frac{\pi x}{L}$$

Eq. (10-42): $$\lambda = \frac{1}{2}\int_0^L \left(\frac{dv}{dx}\right)^2 dx = \frac{\pi^2\delta^2}{4L}$$

Eq. (10-45): $$H = \frac{EA\lambda}{L} = \frac{\pi^2 EA\delta^2}{4L^2} \quad \longleftarrow$$

Eq. (10-46): $$\sigma_t = \frac{H}{A} = \frac{\pi^2 E\delta^2}{4L^2} \quad \longleftarrow$$

(b) ALUMINUM ALLOY

$$E = 10 \times 10^6 \text{ psi} \qquad \sigma_t = 24.67 \times 10^6 \left(\frac{\delta}{L}\right)^2 \text{(psi)}$$

$\frac{\delta}{L}$	$\frac{1}{200}$	$\frac{1}{400}$	$\frac{1}{600}$
σ_t (psi)	617	154	69

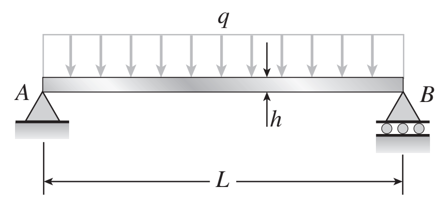
Note: The axial stress increases as the deflection increases.

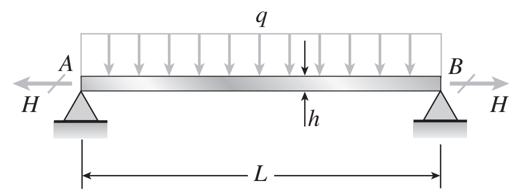
Problem 10.6-2 (a) A simple beam AB with length L and height h supports a uniform load of intensity q (see the *first part* of the figure). Obtain a formula for the curvature shortening λ of this beam. Also, obtain a formula for the maximum bending stress σ_b in the beam due to the load q.

(b) Now assume that the ends of the beam are pinned so that curvature shortening is prevented and a horizontal force H develops at the supports (see the *second part* of the figure). Obtain a formula for the corresponding axial tensile stress σ_t.

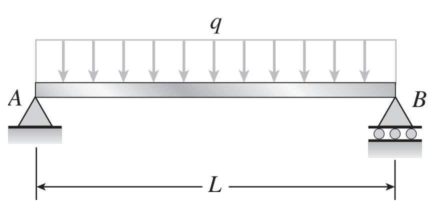
(c) Using the formulas obtained in parts (a) and (b), calculate the curvature shortening λ, the maximum bending stress σ_b, and the tensile stress σ_t for the following steel beam: length $L = 3$ m, height $h = 300$ mm, modulus of elasticity $E = 200$ GPa, and moment of inertia $I = 36 \times 10^6$ mm^4. Also, the load on the beam has intensity $q = 25$ kN/m.

Compare the tensile stress σ_t produced by the axial forces with the maximum bending stress σ_b produced by the uniform load.





Solution 10.6-2 **Beam with uniform load**



h = height of beam

(a) CURVATURE SHORTENING

From Case 1, Table G-2:

$$\frac{dv}{dx} = -\frac{q}{24EI}(L^3 - 6Lx^2 - 4x^3)$$

Eq. (10-42): $$\lambda = \frac{1}{2}\int_0^L \left(\frac{dv}{dx}\right)^2 dx$$

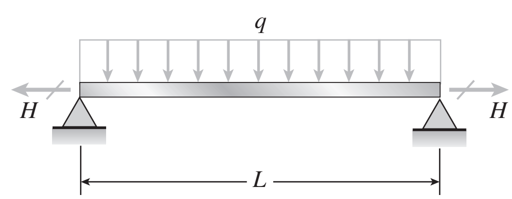
$$= \frac{17q^2L^7}{40{,}320\, E^2I^2} \longleftarrow$$

BENDING STRESS

$$M_{\max} = \frac{qL^2}{8} \qquad c = \frac{h}{2}$$

$$\sigma_b = \frac{Mc}{I} = \frac{qhL^2}{16I} \longleftarrow$$

(b) IMMOVABLE SUPPORTS



Eq. (10-45): $$H = \frac{EA\lambda}{L}$$

Eq. (10-46): $$\sigma_t = \frac{H}{A} = \frac{E\lambda}{L} = \frac{17q^2L^6}{40{,}320\, EI^2} \longleftarrow$$

(c) NUMERICAL VALUES $q = 25$ kN/m

$L = 3$ m $\quad h = 300$ mm $\quad E = 200$ GPa

$I = 36 \times 10^6$ mm^4 $\quad \lambda = 0.01112$ mm $\longleftarrow$

$\sigma_b = 117.2$ MPa $\quad \sigma_t = 0.7411$ MPa $\longleftarrow$

The bending stress is much larger than the axial tensile stress due to curvature shortening.